You have probably noticed that your grapher moves slowly when graphing NINT. This is because it must compute each value as a limit of sums—comparatively slow work even for a microprocessor. Here are some ways to speed up the process:

Change the *x-resolution.* The default resolution is 1, which means that the grapher will compute a function value for every vertical column of pixels. At resolution 2 it computes only every second value, and so on. With higher resolutions, some graph smoothness is sacrificed for speed.

Fundamental Theorem of Calculus

THEOREM 4 The Fundamental Theorem of Calculus, Part 1

If f is continuous on $[a, b]$, then the function

$$F(x) = \int_a^x f(t)\,dt$$

has a derivative at every point x in $[a, b]$, and

$$\frac{dF}{dx} = \frac{d}{dx}\int_a^x f(t)\,dt = f(x).$$

$$\frac{d}{dx}\int_a^x f(t)\,dt = f(x). \qquad (1)$$

EXAMPLE 1 **Applying the Fundamental Theorem**

Find

$$\frac{d}{dx}\int_{-\pi}^{x} \cos t \, dt \quad \text{and} \quad \frac{d}{dx}\int_{0}^{x} \frac{1}{1+t^2}\, dt$$

by using the Fundamental Theorem.

$\cos x$

$\cancel{t}$

$\frac{1}{1+x^2}$

$f(x)$

$\shortparallel$

1. $f(t) = t^3$ $\qquad F(x) = \int_1^x t^3\,dt$

$\frac{1}{4} t^4 \Big|_1^x$

$F(x) = \frac{1}{4}(x^4 - 1) = \frac{1}{4}x^4 - \frac{1}{4}$

$F'(x) = f(x) = x^3$

$$F(x) = \int_1^{x^2} f(t)\,dt$$

$$f(t) = t^3$$

$$F(x) = \int_1^{x^2} t^3\,dt$$

$$\frac{1}{4}t^4 \Big|^{x^2}$$

$$= \frac{1}{4}\left((x^2)^4 - 1\right) = \frac{1}{4}(x^2)^4 - \frac{1}{4}(1)^4$$

$$F'(x) = (x^2)^3 \cdot 2x$$

$$= x^6 \cdot 2x$$

$$= 2x^7$$

$$F'(x) = f(x^2) \cdot 2x$$

$$F(x) = \int_a^{g(x)} f(t)\,dt$$

$$F'(x) = f(g(x)) \cdot g'(x)$$

EXAMPLE 2 The Fundamental Theorem with the Chain Rule

Find dy/dx if $y = \int_1^{x^2} \cos t \, dt$.

$= \cos(x^2) \cdot 2x$

EXAMPLE 3 Variable Lower Limits of Integration

Find dy/dx.

(a) $y = \int_x^5 3t \sin t \, dt$ (b) $y = \int_{2x}^{x^2} \frac{1}{2 + e^t} \, dt$

a) $-\int_5^x 3t \sin t \, dt$

$= -(3x \sin x)$

b) $\int_{2x}^0 \frac{1}{2+e^t} dt + \int_0^{x^2} \frac{1}{2+e^t} dt$

$= -\int_0^{2x} \frac{1}{2+e^t} dt + \int_0^{x^2} \frac{1}{2+e^t} dt$

$= -\frac{1}{2+e^{2x}} \cdot 2 + \frac{1}{2+e^{x^2}} \cdot 2x$

$=$

Example 1

Given $\frac{dy}{dx} = 3x^2 + 4x - 5$ with the initial condition $y(2) = -1$, find $y(3)$.

Method 1:

antiderivative:

$$y = x^3 + 2x^2 - 5x + C$$

$$-1 = 2^3 + 2(2)^2 - 5(2) + C$$

$$y = x^3 + 2x^2 - 5x - 7$$

$$y(3) = 3^3 + 2(3)^2 - 5(3) - 7$$

$$= 23$$

Method 2:

$$-1 + \int_2^3 (3x^2 + 4x - 5)\,dx$$

$$-1 + \left[x^3 + 2x^2 - 5x \,\Big|_2^3\right]$$

$$-1 + 24 = 23$$

EXAMPLE 4 Constructing a Function with a Given Derivative and Value

Find a function $y = f(x)$ with derivative

$$\frac{dy}{dx} = \tan x$$

that satisfies the condition $f(3) = 5$.

$$\int_3^x \tan t \, dt \quad + 5$$

EXPLORATION 2 The Effect of Changing a in $\int_a^x f(t)dt$

The first part of the Fundamental Theorem of Calculus asserts that the derivative of $\int_a^x f(t)\,dt$ is $f(x)$, regardless of the value of a.

1. Graph NDER (NINT (x^2, x, 0, x)).
2. Graph NDER (NINT (x^2, x, 5, x)).
3. Without graphing, tell what the x-intercept of NINT (x^2, x, 0, x) is. Explain.
4. Without graphing, tell what the x-intercept of NINT (x^2, x, 5, x) is. Explain.
5. How does changing a affect the graph of $y = (d/dx)\int_a^x f(t)\,dt$?
6. How does changing a affect the graph of $y = \int_a^x f(t)\,dt$?

THEOREM 4 (continued) **The Fundamental Theorem of Calculus, Part 2**

If f is continuous at every point of $[a, b]$, and if F is any antiderivative of f on $[a, b]$, then

$$\int_a^b f(x)\,dx = F(b) - F(a).$$

This part of the Fundamental Theorem is also called the **Integral Evaluation Theorem.**

EXAMPLE 5 Evaluating an Integral

Evaluate $\int_{-1}^{3} (x^3 + 1)\, dx$ using an antiderivative.

Example 5

A pizza with a temperature of 95°C is put into a 25°C room when $t = 0$. The pizza's temperature is decreasing at a rate of $r(t) = 6e^{-0.1t}$ °C per minute. Estimate the pizza's temperature when $t = 5$ minutes.

Total Area vs Net Area

EXAMPLE 6 Finding Area Using Antiderivatives

Find the area of the region between the curve $y = 4 - x^2$, $0 \leq x \leq 3$, and the x-axis.

How to Find Total Area Analytically

To find the area between the graph of $y = f(x)$ and the x-axis over the interval $[a, b]$ analytically,

1. partition $[a, b]$ with the zeros of f,
2. integrate f over each subinterval,
3. add the absolute values of the integrals.

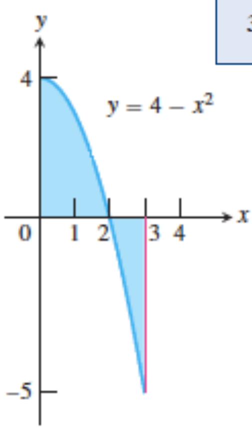
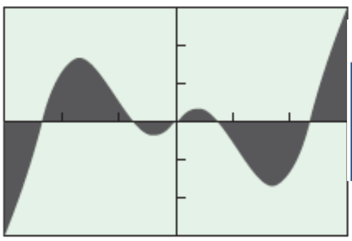


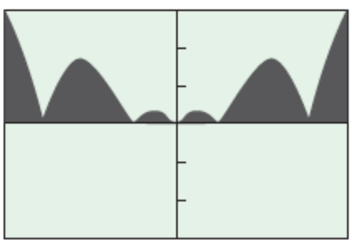
Figure 5.28 The function $f(x) = 4 - x^2$ changes sign only at $x = 2$ on the interval $[0, 3]$. (Example 6)

We can find area numerically by using NINT to integrate the *absolute value* of the function over the given interval. There is no need to partition. By taking absolute values, we automatically reflect the negative portions of the graph across the x-axis to count all area as positive (Figure 5.29).



$[-3, 3]$ by $[-3, 3]$

(a)



$[-3, 3]$ by $[-3, 3]$

(b)

Figure 5.29 The graphs of (a) $y = x \cos 2x$ and (b) $y = |x \cos 2x|$ over $[-3, 3]$. The shaded regions have the same area.

EXAMPLE 7 Finding Area Using NINT

Find the area of the region between the curve $y = x \cos 2x$ and the x-axis over the interval $-3 \le x \le 3$ (Figure 5.29).

How to Find Total Area Numerically

To find the area between the graph of $y = f(x)$ and the x-axis over the interval $[a, b]$ numerically, evaluate

$$\text{NINT}\,(|f(x)|, x, a, b).$$

EXAMPLE 8 Using the Graph of f to Analyze $h(x) = \int_a^x f(t)\,dt$

The graph of a continuous function f with domain [0, 8] is shown in Figure 5.30. Let h be the function defined by $h(x) = \int_1^x f(t)\,dt$.

(a) Find $h(1)$.

(b) Is $h(0)$ positive or negative? Justify your answer.

(c) Find the value of x for which $h(x)$ is a maximum.

(d) Find the value of x for which $h(x)$ is a minimum.

(e) Find the x-coordinates of all points of inflections of the graph of $y = h(x)$.

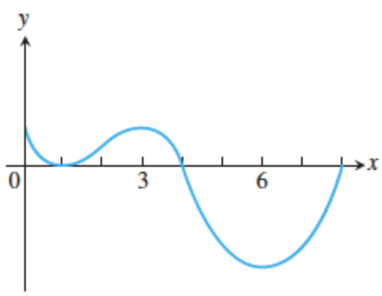
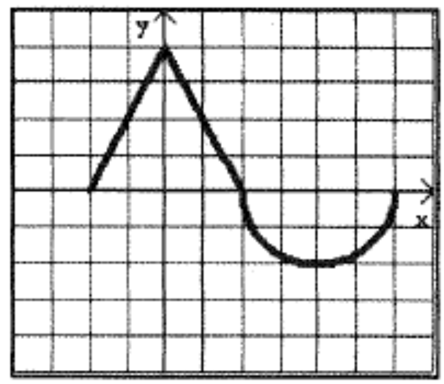


Figure 5.30 The graph of f in Example 8, in which questions are asked about the function $h(x) = \int_1^x f(t)\,dt$.

Example 3

The graph of f' on $-2 \le x \le 6$ consists of two line segments and a semicircle as shown at right. Given that $f(-2) = 5$,

find $f(0)$, $f(2)$, and $f(6)$.



Graph of f'